OBJECTIVE Type Questions

[**1** mark]

Multiple Choice Questions

1. The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is:

(a) 4

(b) ± 4

(c) – 4

(d) 0   [CBSE 2020]

Ans. (b) ± 4

Explanation:

The equation $2x^2 + kx + 2 = 0$ has equal roots, when $D = (k)^2 - 4(2)(2) = 0$

$(\because D = b^2 - 4ac)$

⇒ $k^2 = 16$

or $k = \pm 4$

2. A teacher asks three students to complete the following statement about the nature of the roots of a quadratic equation.

If $q^2 - 4pr > 0$, the roots of the quadratic equation $px^2 + qx + r = 0$ will be ...

Zain answers. "always positive".

Vipul answers, "positive, if p, q, and r are positive".

Suman answers, "negative, if p, q, and r are positive". Who answered correctly?

(a) Zain

(b) Vipul

(c) Suman

(d) None of them

[CBSE Question Bank 2023]

Ans. (c) Suman

Explanation: The given equation is

$$px^2 + qx + r = 0$$

As per the question,

$$q^2 - 4pr > 0$$

The formula to find the roots of the quadratic equation is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Here, $a = p, b = q, c = r$

Case I

Since, $q^2 - 4pr > 0$

$\therefore x = \dfrac{q \pm \sqrt{q^2 - 4pr}}{2}$

Thus, the roots are not always positive.

Case II

Since, $q^2 - 4pr > 0$, also $p, q,$ and r are positive

$\therefore x = \dfrac{-q \pm \sqrt{q^2 - 4pr}}{2p}$

Thus, the roots are not positive.

Case III

Since, $q^2 - 4pr > 0$, also $p, q,$ and r are positive

$\therefore x = \dfrac{-q \pm \sqrt{q^2 - 4pr}}{2p}$

Thus, the roots are negative.
Hence, Suman answered correctly.

3. Ⓐ Which of the following is a quadratic equation?

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$

(b) $-2x^2 = (5 - x)\left(2x - \dfrac{2}{5}\right)$

(c) $(k + 1)x^2 + \dfrac{3}{2}x = 7$, where $k = -1$

(d) $x^3 - x^2 = (x - 1)^3$ [CBSE 2020]

4. For what value of k, $kx^2 + 8x + 2 = 0$ has real roots?

(a) $k < 8$ (b) $k > 8$

(c) $k = 8$ (d) none of these

Ans. (d) none of these

Explanation: $kx^2 + 8x + 2 = 0$

Here, $a = k, b = 8, c = 2$ ($\therefore ax^2 + bx + c = 0$)

For real roots,

$b^2 - 4ac \geq 0$

$\Rightarrow \quad 8^2 - 4 \times k \times 2 \geq 0$

$\Rightarrow \quad 64 - 8k \geq 0$

$\Rightarrow \quad 8 - k \geq 0$

$\Rightarrow \quad k \leq 8$

5. Which of the following equations has 2 as a root?

(a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$

(c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$

[CBSE 2012]

Ans. (c) $2x^2 - 7x + 6 = 0$

Explanation:

(a) Putting $x = 2$ in $x^2 - 4x + 5 = 0$, we get

$(2)^2 - 4(2) + 5 = 0$

$\Rightarrow \quad 4 - 8 + 5 = 0$

$\Rightarrow \quad 1 \neq 0$

So, $x = 2$ is not a root of $x^2 - 4x + 5 = 0$.

(b) Putting $x = 2$ in $x^2 + 3x - 12 = 0$, we get

$(2)^2 + 3(2) - 12 = 0$

$\Rightarrow \quad 4 + 6 - 12 = 0$

$\Rightarrow \quad -2 \neq 0$

So, $x = 2$ is not a root of $x^2 + 3x - 12 = 0$.

(c) Putting $x = 2$ in $2x^2 - 7x + 6 = 0$, we get

$2(2)^2 - 7(2) + 6 = 0$

$\Rightarrow \quad 8 - 14 + 6 = 0$

$\Rightarrow \quad 0 = 0$

So, $x = 2$ is a root of the equation

$2x^2 - 7x + 6 = 0$.

(d) Putting $x = 2$ in $3x^2 - 6x - 2 = 0$, we get

$3(2)^2 - 6(2) - 2 = 0$

$\Rightarrow \quad 12 - 12 - 2 = 0$

$\Rightarrow \quad -2 \neq 0$

So, $x = 2$ is not a root of the equation $3x^2 - 6x - 2 = 0$.

Concept Applied

→ A real number α is said to be the root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

6. A sum of ₹4000 was divided among x persons. Had there been 10 more persons, each would have got ₹80 less. Which of the following represents the above situation?

(a) $x^2 + 10x - 500 = 0$

(b) $8x^2 + 10x - 400 = 0$

(c) $x^2 + 10x + 500 = 0$

(d) $8x^2 + 10x + 400 = 0$

[CBSE Question Bank 2022]

Ans. (a) $x^2 + 10x - 500 = 0$

Explanation: According to the question

If the number of person is x, then each will get

$= \dfrac{4000}{x}$

If 10 person are added then, number of person $= x + 10$,

then each person will get $= \dfrac{4000}{x + 10}$

Now, $\dfrac{4000}{x} - \dfrac{4000}{x + 10} = 80$

$$4000\left(\frac{1}{x}-\frac{1}{x+10}\right)=80$$

$$\left(\frac{1}{x}-\frac{1}{x+10}\right)=\frac{1}{50}$$

$$\frac{10}{x(x+10)}=\frac{1}{50}$$

$\Rightarrow \qquad x^2+10x-500=0$

7. ⓐ Which of the following equations has the sum of its roots as 3?
(a) $2x^2-3x+6=0$
(b) $-x^2+3x-3=0$
(c) $\sqrt{2}\,x^2-\frac{3}{\sqrt{2}}x+1=0$
(d) $3x^2-3x+3=0$ [NCERT Exemplar]

8. Let p be a prime number. The quadratic equation having its roots as factors of p is:
(a) $x^2-px+p=0$
(b) $x^2-(p+1)x+p=0$
(c) $x^2+(p+1)x+p=0$
(d) $x^2-px+p+1=0$
[CBSE SQP Std. 2022]

Ans. (b) $x^2-(p+1)x+p=0$

Explanation: If p is prime number, then it's factors are 1 and p itself.
And it's roots, α and β are it's factors
Hence $\qquad \alpha=1, \beta=p$
$\qquad x=\alpha$ or β
$\qquad 0=(x-\alpha)(x-\beta)$
Which means $=(x-1)(x-p)$
$\qquad =x^2+(-1-p)x+(-1\times-p)$
$\Rightarrow x^2-(p+1)x+p$ is the required quadratic equation.

9. ⓐ Roots of $-x^2+\frac{1}{2}x+\frac{1}{2}=0$ are:

(a) $-\frac{1}{2}, 1$ (b) $\frac{1}{2}, 1$

(c) $-\frac{1}{2}, -1$ (d) $\frac{1}{2}, -\frac{1}{2}$

10. ⓐ Which of the following equations has two distinct real roots?
(a) $2x^2-3\sqrt{2}x+\frac{9}{4}=0$
(b) $x^2+x-5=0$
(c) $x^2+3x+2\sqrt{2}=0$
(d) $5x^2-3x+1=0$ [NCERT Exemplar]

11. The roots of the quadratic equation $2x^2-x-6=0$ are:

(a) $-2, \frac{3}{2}$ (b) $2, -\frac{3}{2}$

(c) $-2, -\frac{3}{2}$ (d) $2, \frac{3}{2}$

Ans. (b) $2, -\frac{3}{2}$

Explanation:
Given, equation is: $2x^2-x-6=0$
$\Rightarrow \qquad 2x^2-4x+3x-6=0$
$\Rightarrow \qquad 2x(x-2)+3(x-2)=0$
$\Rightarrow \qquad (2x+3)(x-2)=0$
$\Rightarrow \qquad x=\frac{-3}{2}, 2$

12. The roots of the equation $x^2-3x-m(m+3)=0$, where m is a constant, are:
(a) $m, m+3$ (b) $-m, m+3$
(c) $m, -(m+3)$ (d) $-m, -(m+3)$
[CBSE 2011]

Ans. (b) $-m, m+3$

Explanation: Given, equation is:
$\qquad x^2-3x-m(m+3)=0$
$\Rightarrow x^2-[(m+3)-m]x-m(m+3)=0$
$\Rightarrow x^2-(m+3)x+mx-m(m+3)=0$
$\Rightarrow x[x-(m+3)]+m[x-(m+3)]=0$
$\Rightarrow \qquad (x+m)[x-(m+3)]=0$
$\Rightarrow \qquad x=-m,(m+3)$

13. If the quadratic equation $mx^2+2x+m=0$, has two equal roots, then the values of m are:
(a) ± 1 (b) $0, 2$
(c) $0, 1$ (d) $-1, 0$
[CBSE 2012]

Ans. (a) ± 1

Explanation:
Given, equation is $mx^2+2x+m=0$
Here, $a=m, b=2$ and $c=m$
So, $\qquad D=b^2-4ac$
$\qquad =(2)^2-4\times m\times m$
$\qquad =4-4m^2$
Since, roots are equal, then
$\qquad D=0$
$\Rightarrow \qquad 4-4m^2=0$
$\Rightarrow \qquad 4=4m^2$
$\Rightarrow \qquad m^2=1$
$\Rightarrow \qquad m=\pm 1$

14. A train covers the distance between Delhi and Agra at a uniform speed. Had the speed been more, it would have taken less time for covering the same distance. To find the

original speed of the train, Amit formed a quadratic equation as $kx(x - 2) + 6 = 0$ and wanted to examine the nature of roots of the quadratic equation so formed.

For equal roots of the quadratic equation, the value(s) of k is:

(a) $k = 6$
(b) $k = 6, -6$
(c) $k = 2, 3$
(d) $k = 0, 3$

Ans. (a) $k = 6$

Explanation: We have

$$kx(x - 2) + 6 = 0$$
$$\Rightarrow kx^2 - 2kx + 6 = 0$$

Here $a = k, b = -2k, c = 6$

For equal roots,

$$b^2 - 4ac = 0$$
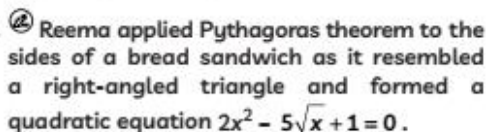$$\Rightarrow (-2k)^2 - 4 \times k \times 6 = 0$$
$$\Rightarrow 4k^2 - 24k = 0$$
$$\Rightarrow 4k(k - 6) = 0$$
$$\Rightarrow k = 0, 6$$

$k = 0$ is rejected as it does not satisfy the condition of a quadratic equation.
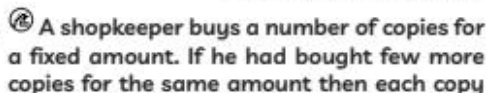
So, the value of k is 6.

15. Reema applied Pythagoras theorem to the sides of a bread sandwich as it resembled a right-angled triangle and formed a quadratic equation $2x^2 - 5\sqrt{x} + 1 = 0$.

The quadratic equation has:

(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

[Mod. NCERT Exemplar]

16. A shopkeeper buys a number of copies for a fixed amount. If he had bought few more copies for the same amount then each copy would have cost him less. To find out how many copies the shopkeeper had bought, Saksham formed a quadratic equation.

$(x^2 + 1)^2 - x^2 = 0$ has:

(a) four real roots
(b) two real roots
(c) no real roots
(d) one real root

[NCERT Exemplar]

17. If $\dfrac{1}{2}$ is a root of the quadratic equation

$x^2 - mx - \dfrac{5}{4} = 0$, then the value of m is:

(a) 2
(b) –2
(c) –3
(d) 3

[British Council 2022]

Ans. (b) – 2

Explanation:

Given, $x = \dfrac{1}{2}$ as root of the equation

$$x^2 - mx - \frac{5}{4} = 0$$

$$\left(\frac{1}{2}\right)^2 - m\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\frac{1}{4} - \frac{m}{2} - \frac{5}{4} = 0$$

$$\frac{m}{2} = \frac{1}{4} - \frac{5}{4}$$

$$\frac{m}{2} = \frac{-4}{4}$$

$$\frac{m}{2} = -1$$

$$m = -2$$

18. Find the nature of the roots for the quadratic equation $x^2 - 3x + 11 = 0$.

(a) No roots
(b) No real roots
(c) Two equal roots
(d) Two distinct real roots

[British Council 2022]

Ans. *(b) No real roots*

Explanation: The given equation is:

$$x^2 - 3x + 11 = 0$$

On comparing it with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -3, c = 11$$

Discriminant, $D = b^2 - 4ac$

$$= (-3)^2 - 4(1)(11)$$
$$= 9 - 44 = -35 < 0$$

Since, the discriminant is negative.

So, the given equation has no real roots.

19. The values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots are:

(a) 8 and 2   (b) 0 and 2

(c) – 8 and 0   (d) 0 and 8

[British Council 2022]

Ans. *(d) 0 and 8*

Explanation: For equal roots, the discriminant must be zero.

Thus, $b^2 - 4ac = 0$

$$k^2 - 8k = 0$$
$$k(k - 8) = 0$$
$$\Rightarrow \qquad k = 0, 8$$

Fill in the Blanks

20. The quadratic equation $2x^2 + px + 3 = 0$ has two equal roots, if $p =$ ....

Ans. $\pm 2\sqrt{6}$

Explanation: $2x^2 + px + 3 = 0$ will have equal roots, when $p^2 - 4(2)(3) = 0$

i.e., $p^2 - 24 = 0,$

$$\Rightarrow \qquad p = \pm \sqrt{24}$$
$$\Rightarrow \qquad p = \pm 2\sqrt{6}$$

21. ⓐ Equation $ax^2 + bx + c = 0$ represents a quadratic equation if and only if ....

22. Sum of roots of the quadratic equation $x^2 - 4x + 2 = 0$ is .... the product of roots.

Ans. twice

Explanation:

$$\text{Sum of roots} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= -\frac{(-4)}{1} = 4$$

$$\text{Product of roots} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{2}{1} = 2$$

So, sum of roots = 2 (product of roots)

23. ⓐ The quadratic equation $2x^2 + x + 4 = 0$ has .... real roots.

24. The roots of $x + \dfrac{1}{x} = 2$ are ....

Ans. 1, 1

Explanation: The given equation can be written as

$$x^2 - 2x + 1 = 0,$$

or $(x - 1)^2 = 0$

So, roots are 1, 1.

25. The sum of the roots of the quadratic equation $2x^2 + 14x + 24 = 0$ is ....

Ans. –7

Explanation:

$$\text{Sum of roots} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-14}{2}, \text{ i.e., } -7$$

Assertion Reason

Direction for questions 26 to 30: In question number 26 to 30, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

26. Assertion (A): The equation $x^2 + 3x + 1 = (x - 2)^2$ is a quadratic equation.

Reason (R): Any equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$ is called a quadratic equation.

Ans. *(d) Assertion (A) is false but reason (R) is true.*

Explanation: We have,

$$x^2 + 3x + 1 = (x - 2)^2$$
$$\Rightarrow \quad x^2 + 3x + 1 = x^2 - 4x + 4$$
$$\Rightarrow \qquad 7x - 3 = 0,$$

It is not of the form $ax^2 + bx + c = 0$

We know that, the equation is of the form $ax^2 + bx + c = 0$, where $a \neq 0$ is called a quadratic equation.

Hence, assertion is false but the reason is true.

27. ⓐ Assertion (A): Sum and product of roots of

$$2x^2 - 3x + 5 = 0 \text{ are } \frac{3}{2} \text{ and}$$

$$\frac{5}{2} \text{ respectively.}$$

Reason (R): If α and β are the roots of $ax^2 + bx + c = 0$, then sum of roots $= \alpha + \beta = \dfrac{-b}{a}$ and product of roots $= \alpha\beta = \dfrac{c}{a}$.

28. Assertion (A): The value of $k = 2$, if one root of the quadratic equation $6x^2 - x - k = 0$ is $\dfrac{2}{3}$.

Reason (R): The quadratic equation $ax^2 - bx + c = 0$, $a \neq 0$ has two roots.

Ans. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Explanation: As one root is $\dfrac{2}{3}$

$$\Rightarrow \qquad x = \dfrac{2}{3}$$

$$6 \times \left(\dfrac{2}{3}\right)^2 - \dfrac{2}{3} - k = 0$$

$$6 \times \dfrac{4}{9} - \dfrac{2}{3} = k$$

$$k = \dfrac{8}{3} - \dfrac{2}{3} = \dfrac{6}{3} = 2$$

$$k = 2$$

We know that, a quadratic equation has two roots.

Hence, both assertion and reason are true but reason is not the correct explanation of the assertion.

29. Assertion (A): The roots of the quadratic equation $x^2 - 2x + 2 = 0$ are imaginary.

Reason (R): If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

Ans. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Explanation: The given equation is,

$$x^2 + 2x + 2 = 0$$

Discriminant, $D = b^2 - 4ac$

$$= (2)^2 - 4 \times 1 \times 2$$

$$= 4 - 8 = -4 < 0$$

Thus, roots are imaginary.

Hence, both assertion and reason are true and reason is the correct explanation of the assertion.

30. Assertion (A): The equation $8x^2 + 3kx + 2 = 0$ has equal roots then the value of k is $\pm \dfrac{8}{3}$.

Reason (R): The equation $ax^2 + bx + c = 0$ has equal roots if $D = b^2 - 4ac = 0$.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

31. A ship is sailing very close to a light house and a right-angled triangle is formed by the light house, by the line joining the ship to the base of the light house and by the line joining the ship to the top of the light house. At a particular instant, the hypotenuse of the right-angled triangle so formed is 4 m more than thrice the shortest side and the third side is 1 m less than the hypotenuse.

(A) Taking the length of the shortest side of the triangle as x m, form the quadratic equation of the given situation.

(B) Find the length of the shortest side of the right-angled triangle formed.

(C) Find the length of the third side of the right-angled triangle formed.

Ans. (A) Here, the length of the shortest side of the right-angled triangle $= x$ m.

According to the question,

Hypotenuse $= (3x + 4)$ m

and the third side $= (3x + 3)$ m.

Applying Pythagoras theorem,

$$(3x + 4)^2 = (3x + 3)^2 + x^2$$

$$\Rightarrow \quad 9x^2 + 24x + 16 = 9x^2 + 18x + 9 + x^2$$

Simplifying further, we get

$$x^2 - 6x - 7 = 0$$

Hence, the required quadratic equation is
$$x^2 - 6x - 7 = 0.$$

32. *The two gears shown in the figure resemble two circles which touch each other externally. The sum of their areas is 130π sq. cm and distance between their centres is 14 cm.*

(A) Taking the radius of one circle to be x cm, the quadratic equation for the situation described above is:
(a) $x^2 - 16x + 63 = 0$ (b) $x^2 - 16x - 63 = 0$
(c) $x^2 - 14x + 33 = 0$ (d) $x^2 + 14x - 33 = 0$

(B) ✎ The radii of the two circles are:
(a) 3 cm and 11 cm (b) 5 cm and 9 cm
(c) 4 cm and 10 cm (d) 6 cm and 8 cm

(C) The roots of the quadratic equation $abx^2 + (b^2 - ac)x - bc = 0$ are:
(a) real and equal
(b) real and distinct
(c) no real roots exist
(d) none of the above

(D) ✎ The roots of the quadratic equation $a^2x^2 - 3abx + 2b^2 = 0$ are:
(a) $\dfrac{2b}{a}, \dfrac{2b}{a}$ (b) $\dfrac{b}{a}, \dfrac{b}{a}$
(c) $-\dfrac{b}{a}, -\dfrac{2b}{a}$ (d) $\dfrac{b}{a}, \dfrac{2b}{a}$

(E) The value of p for which the quadratic equation $x(x - 4) + p = 0$ has real and distinct roots, is:
(a) $p < 4$ (b) $p > 4$
(c) $p < -4$ (d) $p > -4$

Ans. (A) (c) $x^2 - 14x + 33 = 0$

Explanation: Here, the radius of one circle
$= x$ cm.
Then the radius of the second circle
$= (14 - x)$ cm.
The area of the first circle $= \pi x^2$ cm^2
and radius of the second circle
$= \pi(14 - x)^2$ cm^2.

According to the question,
$$\pi x^2 + \pi(14 - x)^2 = 130\pi$$
$$\Rightarrow \pi(x^2 + 196 - 28x + x^2) = 130\pi$$
Simiplifying further,
$$2x^2 - 28x + 196 = 130$$
$$\Rightarrow \quad 2x^2 - 28x + 66 = 0$$
$$\Rightarrow \quad x^2 - 14x + 33 = 0$$

(C) (b) real and distinct
Explanation: To find the nature of roots of the given quadratic equation, we have to find the value of the discriminant D.
$$D = B^2 - 4AC$$
$$= (b^2 - ac)^2 - 4(ab)(-bc)$$
$$= b^4 + a^2c^2 - 2ab^2c + 4ab^2c$$
$$= b^4 + a^2c^2 + 2ab^2c$$
$$= (b^2)^2 + (ac)^2 + 2 \times b^2 \times ac$$
$$= (b^2 + ac)^2,$$
which is greater than zero
∴ The roots of given quadratic equation are real and distinct.

(E) (a) $p < 4$
Explanation: The given equation can be written as, $x^2 - 4x + p = 0$.
Now, for real and distinct roots, $D > 0$.
$$\Rightarrow \quad (-4)^2 - 4(1)(p) > 0$$
$$\Rightarrow \quad 16 > 4p$$
$$\Rightarrow \quad p < 4.$$

33. *A chess board contains 64 equal squares and the area of each square is 6.25 cm². A border round the board is 3 cm wide.*

(A) Find the area of the chess board.
(B) ✎ Find the length of the side of the chess board.
(C) ✎ If $x = \dfrac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

Ans. (A)
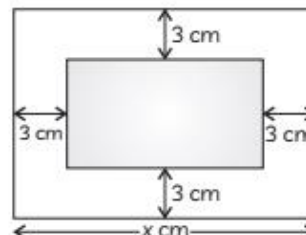
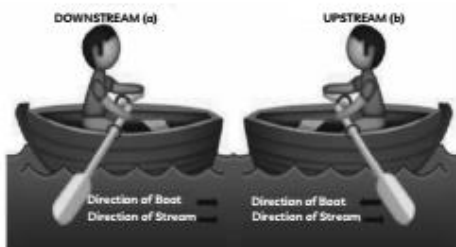Therefore, as the number of small squares = 64 and area of each square = 6.25 cm². Area of the chess board (excluding the border) = 64 × 6.25 cm²

= 400 cm².

34. The speed of a motor boat is 20 km/hr. For covering the distance of 15 km, the boat took 1 hour more for upstream than downstream.

DOWNSTREAM (a)   UPSTREAM (b)

Direction of Boat
Direction of Stream   Direction of Boat
Direction of Stream

(A) Let speed of the stream be x km/hr. Then speed of the motorboat in upstream will be:

(a) 20 km/hr   (b) $(20 + x)$ km/hr

(c) $(20 - x)$ km/hr   (d) 2 km/hr

(B) What is the relation between speed, distance and time?

(a) speed = (distance)/time

(b) distance = (speed)/time

(c) time = speed × distance

(d) speed = distance × time

(C) ㉔ Which is the correct quadratic equation for the speed of the current?

(a) $x^2 + 30x - 200 = 0$

(b) $x^2 + 20x - 400 = 0$

(c) $x^2 + 30x - 400 = 0$

(d) $x^2 - 20x - 400 = 0$

(D) What is the speed of current?

(a) 20 km/hour   (b) 10 km/hour

(c) 15 km/hour   (d) 25 km/hour

(E) ㉔ How much time boat took in down-stream?

(a) 90 minutes   (b) 15 minutes

(c) 30 minutes   (d) 45 minutes

[CBSE Question Bank 2022]

Ans. (A) (c) $(20 - x)$ km/hr

Explanation: Here, speed of motor boat

= 20 km/h.

Let, the speed of stream = x km/hr

∴ Speed of boat in upstream $(20 - x)$ km/hr

(B) (a) $Speed = \dfrac{Distance}{Time}$

(D) (b) 10 km/hr

Explanation: Simplifying,

$$x^2 + 30x - 400 = 0.$$

⇒ $x^2 + 40x - 10x - 400 = 0$

⇒ $x(x + 40) - 10(x + 40) = 0$

⇒ $(x - 10)(x + 40) = 0$

⇒ $x = 10, -40$

∴ $x = 10$

[∵ speed cannot be negative]

Hence, speed of the stream is 10 km/hr.

35. A passenger while boarding a plane slipped from the stairs and got hurt. The pilot took the passenger to the emergency clinic at the airport for treatment. Due to this, plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/h than the usual speed.

On the basis of above information answer the following questions:

(A) ㉔ If x represent the usual speed of the plane, then find the quadratic equation in x, formed in this case.

(B) What is the usual speed of the plane?

(C) Find the increased speed of the plane.

Ans. (B) The quadratic equation is

$x^2 + 250x - 750000$ [From A]

$x^2 + 1000x - 750x - 750000 = 0$

$(x + 1000)(x - 750) = 0$

$x = 750, x = -1000$ (speed can't be negative)

So, speed of plane is 750 km/hr.

(C) According to the question, speed of the plane is increased by 250 km/hr

∴ $x + 250 = 750 + 250$ [from (B), $x = 750$]

So, the increased speed of the plane is 1000 km/hr.

VERY SHORT ANSWER Type Questions (VSA)

[**1** mark]

36. Find the values of 'k' for which $x = 2$ is a solution of the equation $kx^2 + 2x - 3 = 0$. [CBSE 2019]

Ans. Given equation is, $kx^2 + 2x - 3 = 0$

If $x = 2$, then
$$k(2)^2 + 2(2) - 3 = 0$$
$$4k + 4 - 3 = 0$$
$$\Rightarrow \qquad 4k = -1$$
$$\Rightarrow \qquad k = -\frac{1}{4}$$

Hence, the value of k is $-\frac{1}{4}$.

37. Find the value(s) of k for which the quadratic equation $3x^2 + kx + 3 = 0$ has real and equal roots. [CBSE 2019]

Ans. Given, quadratic equation is: $3x^2 + kx + 3 = 0$

For real and equal roots, $b^2 - 4ac = 0$

Here, $a = 3, b = k$ and $c = 3$
$$\therefore \qquad b^2 - 4ac = (k)^2 - 4 \times 3 \times 3 = 0$$
$$\Rightarrow \qquad k^2 = 36$$
$$\Rightarrow \qquad k = \pm 6$$

Hence, the values of k are ± 6.

38. ⓐ For what values of k does the quadratic equation $4x^2 - 12x - k = 0$ has no real roots? [CBSE 2015]

39. Is 0.2 a root of the equation $x^2 - 0.4 = 0$? Justify. [NCERT Exemplar]

Ans. No.

As, $\qquad x^2 - 0.4 = (0.2)^2 - 0.4$
$$= 0.04 - 0.4$$
$$= -0.36 \neq 0$$

So, 0.2 is not a root of $x^2 - 0.4 = 0$.

40. For what values of k, are the roots of the equation $x^2 + 4x + k = 0$ real? [CBSE 2012]

Ans. Since, the roots of the equation $x^2 + 4x + k = 0$ are real,
$$\therefore \qquad D \geq 0$$
$$b^2 - 4ac \geq 0$$
Here, $\qquad a = 1, b = 4, c = k$
$$\Rightarrow \qquad (4)^2 - 4 \times 1 \times k \geq 0$$
$$\Rightarrow \qquad 16 - 4k \geq 0$$
$$\Rightarrow \qquad k \leq 4$$

Hence, for all $k \leq 4$, the given equation has real roots.

41. If $x = 2$ and $m = 3$, the equation is $3x^2 - 2kx + 2m = 0$, find k. [Diksha]

Ans. Given equation is,
$$3x^2 - 2kx + 2m = 0$$
Since, $\qquad x = 2$ and $m = 3$ [Given]
So, $\quad 3(2)^2 - 2k(2) + 2(3) = 0$
$$\Rightarrow \qquad 12 - 4k + 6 = 0$$
$$\Rightarrow \qquad -4k + 18 = 0$$
$$\Rightarrow \qquad k = \frac{9}{2}.$$

42. ⓐ If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of 'k'. [CBSE 2017]

43. For what value(s) of 'a', the quadratic equation $30ax^2 - 6x + 1 = 0$ has no real roots? [CBSE SQP 2020]

Ans. Given, quadratic equation is
$$30ax^2 - 6x + 1 = 0$$
On comparing this equation with
$$Ax^2 + Bx + C = 0,$$
we get $A = 30a, B = -6$ and $C = 1$

For no real roots,
$$D < 0$$
i.e. $\qquad B^2 - 4AC < 0$
$$(-6)^2 - 4 \times (30a) \times (1) < 0$$
$$36 < 120a$$
or $\qquad 10a > 3$
or $\qquad a > 0.3$

Hence, the value of a should be greater than 0.3 for unreal roots.

44. Find the value of k for which the roots of the quadratic equation $2x^2 + kx + 8 = 0$ will have equal value. [CBSE 2012]

Ans. Given: quadratic equation is $2x^2 + kx + 8 = 0$

For roots of the equation to be equal,
$$D = 0$$
i.e., $\qquad b^2 - 4ac = 0$
Here, $a = 2, b = k$ and $c = 8$
$$\Rightarrow \qquad k^2 - 4 \times 2 \times 8 = 0$$
$$\Rightarrow \qquad k^2 = 64$$
$$\Rightarrow \qquad k = \pm 8$$

Hence, the value of k is 8 or −8.

45. Calculate the roots of the quadratic equation $x^2 + 5x - (\alpha + 1)(\alpha + 6) = 0$, where α is a constant. [CBSE 2011]

Ans. Given, quadratic equation is :

$$x^2 + 5x - (\alpha + 1)(\alpha + 6) = 0$$
$$\Rightarrow \quad x^2 + (\alpha + 6)x - (\alpha + 1)x - (\alpha + 1)(\alpha + 6) = 0$$
$$\Rightarrow \quad x[x + (\alpha + 6)] - (\alpha + 1)[x + (\alpha + 6)] = 0$$
$$\Rightarrow \qquad [x + (\alpha + 6)][x - (\alpha + 1)] = 0$$
$$\Rightarrow \quad x + (\alpha + 6) = 0 \text{ or } x - (\alpha + 1) = 0$$
$$\Rightarrow \qquad x = -(\alpha + 6) \text{ or } x = (\alpha + 1)$$

46. **If the quadratic equation** $px^2 - 2\sqrt{5}\, px + 15 = 0$, **has two equal roots, then find the value of** p. [CBSE 2015]

47. Shweta went to the market to buy a piece of cloth to make a frock for herself. The piece of cloth costs a sum of rupees. If the piece was few metres longer, then each meter of cloth would cost few amount of rupees less, but the cost of the piece would remained unchanged. She formulated a quadratic equation as $9x^2 + 6kx + 4 = 0$ to get the exact length of the piece and also the original cost of the piece of cloth.

For what values of k, the quadratic equation has equal roots? [Mod. CBSE SQP 2020]

Ans. Given, equation is

$$9x^2 + 6kx + 4 = 0$$

On comparing this equation with

$$ax^2 + bx + c = 0;$$

We get, $a = 9, b = 6k$ and $c = 4$

For equal roots,

$$b^2 - 4ac = 0$$
$$(6k)^2 - 4 \times 9 \times 4 = 0$$
$$\Rightarrow \quad 6 \times 6 \times k^2 = 4 \times 9 \times 4$$
$$\Rightarrow \qquad k^2 = 2 \times 2 = 4$$
$$\Rightarrow \qquad k = \pm 2$$

Hence, the values of k are ± 2.

48. A school planned a picnic. The budget for food was limited and fixed. But some students failed to go and thus the cost of food for each member increased by a certain amount. In order to find out the number of students who attended the picnic, Sneha formed a quadratic equation $2x^2 - 4x + 3 = 0$.

Find the nature of the roots of the above quadratic equation. [Mod. CBSE 2011]

Ans. Given quadratic equation is:

$$2x^2 - 4x + 3 = 0$$

Here, $a = 2, b = -4$ and $c = 3$

Discriminant, $D = b^2 - 4ac$

$$= (-4)^2 - 4 \times 2 \times 3$$
$$= 16 - 24 = -8 < 0$$
$$\therefore \qquad D < 0$$

Hence, the given equation does not have real roots.

49. Samsher went to the market to buy pure ghee for a function in his house. But he was surprised to find that the cost of ghee per kilogram had increased by a certain amount. With the increased price per kilogram, he formulated a quadratic equation $x^2 - ax + 1 = 0$ to calculate the original price of ghee, as now it would be possible to purchase less quantity of ghee for the same amount.

For what values of 'a', does the quadratic equation not have real roots?

[Mod. CBSE 2016]

Ans. Given quadratic equation is $x^2 - ax + 1 = 0$.

On comparing the given equation with

$$Ax^2 + Bx + C = 0, \text{ we get}$$
$$A = 1, B = -a \text{ and } C = 1$$

For no real roots, $D < 0$

$$\Rightarrow \qquad B^2 - 4AC < 0$$

i.e., $(-a)^2 - 4 \times 1 \times 1 < 0$

$\Rightarrow$ $\qquad a^2 < 4$

or $\qquad a < \pm\sqrt{4}$

or $\qquad -2 < a < 2$

Hence, the value of 'a' lies between -2 and 2.

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

50. For what positive values of k, does the quadratic equation $3x^2 - kx + 3 = 0$ not have real roots ? **[CBSE 2017]**

Ans. Given: quadratic equation $3x^2 - kx + 3 = 0$, has no real roots.

On comparing the given equation with $ax^2 + bx + c = 0$, we have:

$a = 3$, $b = -k$, $c = 3$

Then, discriminant,

$D = b^2 - 4ac$

$= (-k)^2 - 4 \times 3 \times 3$

$= k^2 - 36$

But for no real roots, $D < 0$

Then $\qquad k^2 - 36 < 0$

$\Rightarrow \qquad k^2 < 36$

$\Rightarrow \qquad k < \pm 6$

$\Rightarrow \qquad k > -6$ or $k < 6$

Hence, for all $-6 < k < 6$, the given equation has no real roots.

51. Solve for x : $6x^2 + 11x + 3 = 0$ **[CBSE 2018]**

Ans. $\qquad 6x^2 + 11x + 3 = 0$

$\Rightarrow \qquad 6x^2 + 9x + 2x + 3 = 0$

$\Rightarrow \qquad 3x(2x+3) + 1(2x+3) = 0$

$\Rightarrow \qquad (2x+3)(3x+1) = 0$

$\Rightarrow \qquad 2x + 3 = 0$ or $3x + 1 = 0$

i.e., $\qquad x = -\dfrac{3}{2}$ or $x = -\dfrac{1}{3}$

52. ⓐ Solve for x : $8x^2 - 2x - 3 = 0$ **[CBSE 2020]**

53. Solve the following quadratic equation:

$6a^2x^2 - 7abx - 3b^2 = 0$ **[CBSE 2017]**

Ans. Given: quadratic equation is :

$6a^2x^2 - 7abx - 3b^2 = 0$

On comparing the given equation with $Ax^2 + Bx + C = 0$, we get

$A = 6a^2$, $B = -7ab$ and $C = -3b^2$

Then, discriminant,

$D = B^2 - 4AC$

$= (-7ab)^2 - 4 \times 6a^2 \times (-3b^2)$

$= 49a^2b^2 + 72a^2b^2$

$= 121\,a^2b^2$

Then, roots of the equation, $x = \dfrac{-B \pm \sqrt{D}}{2A}$

$= \dfrac{-(-7ab) \pm \sqrt{121a^2b^2}}{2 \times 6a^2}$

$= \dfrac{7ab \pm 11ab}{12a^2}$

$= \dfrac{7ab + 11ab}{12a^2}$ or $\dfrac{7ab - 11ab}{12a^2}$

$= \dfrac{18ab}{12a^2}$ or $\dfrac{-4ab}{12a^2}$

$= \dfrac{3b}{2a}$ or $\dfrac{-b}{3a}$

Hence, the roots of the given equation are: $\dfrac{3b}{2a}$

and $\dfrac{-b}{3a}$.

54. Solve for x: $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

[CBSE 2010]

Ans. Given: $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

On comparing the above equation with $ax^2 + bx + c = 0$, we get

$a = \sqrt{3}$, $b = 10$ and $c = -8\sqrt{3}$

Then, discriminant,

$D = b^2 - 4ac$

$= (10)^2 - 4 \times \sqrt{3} \times (-8\sqrt{3})$

$= 100 + 96$

$= 196$

Roots of equation are given by,

$x = \dfrac{-b \pm \sqrt{D}}{2a}$

$= \dfrac{-10 \pm \sqrt{196}}{2 \times \sqrt{3}}$

$= \dfrac{-10 \pm 14}{2\sqrt{3}}$

$= \dfrac{4}{2\sqrt{3}}$ and $-\dfrac{24}{2\sqrt{3}}$

$= \dfrac{2}{\sqrt{3}}$ and $-\dfrac{12}{\sqrt{3}}$

Rationalising the denominator

$$= \frac{2\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \text{ and } \frac{-12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3} \text{ and } -4\sqrt{3}$$

Hence, the roots of the given equation are $\frac{2\sqrt{3}}{3}$ and $-4\sqrt{3}$.

55. Solve the quadratic equation

$x^2 + 2\sqrt{2}x - 6 = 0$ for x.

Ans. $x^2 + 2\sqrt{2}x - 6 = 0$

Splitting the middle term, we get

$\Rightarrow \quad x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$

$x^2 + 3\sqrt{2}x - \sqrt{2}x - \sqrt{2}\times3\sqrt{2} = 0$

$x(x+3\sqrt{2}) - \sqrt{2}(x+3\sqrt{2}) = 0$

$(x+3\sqrt{2})(x-\sqrt{2}) = 0$

$x+3\sqrt{2} = 0,\ x-\sqrt{2} = 0$

$x = -3\sqrt{2} \text{ or } x = \sqrt{2}$

Therefore,

Roots of the given quadratic equation:

$x = -3\sqrt{2}, x = \sqrt{2}$

56. Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.
[NCERT Exemplar]

Ans. Yes, there exists a quadratic equation whose coefficients are rational but both of its roots are irrational.

Consider the quadratic equation

$x^2 - 6x + 7 = 0$

Here, $D = b^2 - 4ac$

$= (-6)^2 - 4(1)(7)$

$\Rightarrow D = 36 - 28 = 8$

Since, discriminant is not a perfect square, therefore it will have irrational roots.

The roots will be

$$\frac{-b\pm\sqrt{D}}{2a} = \frac{6\pm\sqrt{8}}{2\times1} = \frac{6\pm2\sqrt{2}}{2}$$

The roots will be $3\pm\sqrt{2}$ i.e. $3 + \sqrt{2}$ and $3 - \sqrt{2}$ which are irrational.

57. Find the nature of roots of the quadratic equation $4x^2 - 12x + 9 = 0$.

Ans. Given equation is $4x^2 - 12x + 9 = 0$

∴ $a = 4, b = -12, c = 9$

We know,

Discriminant $= b^2 - 4ac$

$= (-12)^2 - 4\times4\times9$

$= 144 - 144$

$= 0$

Hence, the nature of roots is real and equal.

58. Solve for x:

$\frac{x+3}{x+2} = \frac{3x-7}{2x-3}, x \ne 2, \frac{3}{2}$
[CBSE 2013]

Ans. $\Rightarrow \quad (x+3)(2x-3) = (x+2)(3x-7)$

$\Rightarrow \quad 2x^2 + 6x - 3x - 9 = 3x^2 + 6x - 7x - 14$

$\Rightarrow 2x^2 - 3x^2 + 3x + x - 9 + 14 = 0$

$\Rightarrow \quad -x^2 + 4x + 5 = 0$

$\Rightarrow \quad x^2 - 4x - 5 = 0$

$\Rightarrow \quad x^2 - 5x + x - 5 = 0$

(on splitting the middle term)

$\Rightarrow \quad x(x-5) + 1(x-5) = 0$

$\Rightarrow \quad (x+1)(x-5) = 0$

$\Rightarrow \quad x = -1, 5$

Hence, the values of x are –1 and 5.

59. Find the roots of the quadratic equation

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.
[CBSE 2017]

60. If $b = 0$ and $c < 0$, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify. **[NCERT Exemplar]**

Ans. The given quadratic equation is:

$x^2 + bx - c = 0$

$x^2 + (0)x - (-c) = 0$

$x^2 + c = 0$

$x = \pm\sqrt{c}$

Thus, we can say roots are numerically equal and opposite in sign.

61. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots. **[CBSE 2016]**

62. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$ then find the values of a and b. **[CBSE 2016]**

Ans. Since $x = \frac{2}{3}$ and $x = -3$ are the roots of the quadratic equation $ax^2 + 7x + b = 0$.

So, sum of roots:

$\frac{2}{3} + (-3) = -\frac{7}{a}\ \frac{(-\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

$\Rightarrow \quad \frac{-7}{3} = \frac{-7}{a}$

$\Rightarrow \quad a = 3.$

Also, product of roots:

$\frac{2}{3}\times(-3) = \frac{b}{a}\ \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$

$\Rightarrow \quad -2 = \frac{b}{3} \quad [-a = 3]$

$\Rightarrow \quad b = -6$

Hence, the values of a and b are 3 and –6 respectively.

63. If Ritu were younger by 5 years than what she really is, then the square of her age would have been 11 more than five times her present age. What is her present age?

[CBSE Term-2 SQP 2022]

Ans. Let the present age of Ritu be x years.

$$(x-5)^2 = 5x + 11$$
$$x^2 - 15x + 14 = 0$$
$$(x-14)(x-1) = 0$$
$$\Rightarrow \qquad x = 1 \text{ or } 14$$
$$x = 14 \text{ years}$$

(rejecting $x = 1$ as in that case Ritu's age 5 years ago will be – ve)

[CBSE Marking Scheme Term-2 SQP 2022]

Let, the present age of Ritu be x years.
Then, according to the question

$$(x-5)^2 = 11 + 5x$$
$$\Rightarrow \qquad x^2 - 10x + 25 = 11 + 5x$$
$$\Rightarrow \qquad x^2 - 15x + 14 = 0$$
$$\Rightarrow \qquad x^2 - 14x - x + 14 = 0$$
$$\Rightarrow \qquad x(x-14) - 1(x-14) = 0$$
$$\Rightarrow \qquad (x-1)(x-14) = 0$$
$$\Rightarrow \qquad x = 1, 14$$

But $x = 1$ is not possible, as in this case, $(x - 5)$ will be negative.

$$\therefore \qquad x = 14$$

Hence, the present age of Ritu is 14 years.

64. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

[Delhi Gov. QB 2022, CBSE 2015]

Ans. Let the original average speed (first) of train be x km./h.

$$\therefore \quad \frac{54}{x} + \frac{63}{x+6} = 3$$
$$\Rightarrow \quad 54x + 324 + 63x = 3x(x+6)$$
$$\Rightarrow \quad x^2 - 33x - 108 = 0$$

Solving to get $x = 36$

$\therefore$ First speed of train = 36 km/h.

[CBSE Marking Scheme 2015]

65. If a and b are the roots of the equation $x^2 + ax - b = 0$, then find a and b. [Diksha]

Ans. Given quadratic equation is:

$$x^2 + ax - b = 0$$

Sum of the roots $= a + b$

$$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} = -a$$

Product of roots $= ab$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2} = -b$$

So, $\qquad a + b = -a$
$$\Rightarrow \qquad b = -2a$$
and, $\qquad ab = -b$
$$\Rightarrow \qquad a = -1$$

Putting $a = -1$ in $b = -2a$, we get

$$b = -2 \times (-1) = 2$$

Hence, $a = -1$ and $b = 2$.

66. Find the value of m so that the quadratic equation $mx(5x - 6) + 9 = 0$ has two equal roots. [CBSE Term-2 SQP 2022]

Ans.
$$5mx^2 - 6mx + 9 = 0$$
$$b^2 - 4ac = 0$$
$$\Rightarrow \qquad (-6m)^2 - 4(5m)(9) = 0$$
$$\Rightarrow \qquad 36m(m-5) = 0$$
$$\Rightarrow \qquad m = 0, 5;$$
Rejecting $\qquad m = 0$
we get $\qquad m = 5$

[CBSE Marking Scheme Term-2 SQP 2022]

Given quadratic equation is:

$$mx(5x - 6) + 9 = 0$$
$$\Rightarrow \qquad 5mx^2 - 6mx + 9 = 0$$

Since, the equation has equal roots,

$$\therefore \qquad \text{Discriminant} = 0$$
$$\Rightarrow \qquad b^2 - 4ac = 0$$

Here, $a = 5m$, $b = -6m$, $c = 9$

$$\therefore \qquad (-6m)^2 - 4 \times 5m \times 9 = 0$$
$$\Rightarrow \qquad 36m^2 - 180m = 0$$
$$\Rightarrow \qquad 36m(m-5) = 0$$
$$\Rightarrow \qquad m = 0, 5$$

But, $m \neq 0$, as it does not satisfy the given equation.

$$\therefore \qquad m = 5$$

Hence, the value of m is 5.

⚠️ **Caution**

↪ *Student should remember that the standard form of a quadratic equation is $ax^2 + bx + c = 0$, $a \neq 0$.*

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

67. ⓐFind the roots of the quadratic equations by using the quadratic formula in each of the following: [NCERT]

(A) $5x^2 + 13x + 8 = 0$

(B) $-3x^2 + 5x + 12 = 0$

(C) $-x^2 + 7x - 10 = 0$

(D) $x^2 + 2\sqrt{2}x - 6 = 0$

[CBSE 2017, 16, 15, 14, 11]

(E) $x^2 - 3\sqrt{5}x + 10 = 0$

[CBSE 2017, 16, 15, 14, 11]

(F) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

[CBSE 2017, 16, 15, 14, 11]

68. Find the roots of the following quadratic equations by the factorisation method:

(A) $2x^2 + \frac{5}{3}x - 2 = 0$

(B) ⓐ $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

(C) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ [CBSE 2016, 12]

(D) ⓐ $3x^2 + 5\sqrt{5}x - 10 = 0$

[CBSE 2016, 12]

(E) ⓐ $21x^2 - 2x + \frac{1}{21} = 0$ [NCERT Exemplar]

Ans. (A) The given equation is: $2x^2 + \frac{5x}{3} - 2 = 0$

Multiplying both the sides by 3, we get

$6x^2 + 5x - 6 = 0$

Splitting the middle term, we have

$6x^2 + 9x - 4x - 6 = 0$

$\Rightarrow 3x(2x + 3) - 2(2x + 3) = 0$

$\Rightarrow \qquad (2x + 3)(3x - 2) = 0$

$\Rightarrow \qquad 2x + 3 = 0 \quad \text{or} \quad 3x - 2 = 0$

$x = -\frac{3}{2} \text{ or } x = \frac{2}{3}$

Hence, the roots of the given equation are $-\frac{3}{2}$ and $\frac{2}{3}$.

(C) The given equation is: $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

Splitting the middle term, we have

$3\sqrt{2}x^2 - 6x + x - \sqrt{2} = 0$

$\Rightarrow 3\sqrt{2}x^2 - 3\sqrt{2}.\sqrt{2}x + x - \sqrt{2} = 0$

$\Rightarrow \quad 3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$

$\Rightarrow \qquad (x - \sqrt{2})(3\sqrt{2}x + 1) = 0$

$\Rightarrow \qquad\qquad x - \sqrt{2} = 0$

or $\qquad\qquad 3\sqrt{2}x + 1 = 0$

$\Rightarrow \qquad\qquad x = \sqrt{2} \text{ or } x = \frac{-1}{3\sqrt{2}} = \frac{-\sqrt{2}}{6}$

Hence, the roots of the equation are $\sqrt{2}$ and $\frac{-\sqrt{2}}{6}$.

69. Solve for x:

$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}, x \neq 1, -2, 2$

Ans. We have,

$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$

$\Rightarrow \frac{(x+1)(x+2) + (x-2)(x-1)}{(x-1)(x+2)}$

$= \frac{4(x-2) - (2x+3)}{x-2}$

$\Rightarrow \frac{(x^2+3x+2) + (x^2-3x+2)}{x^2+x-2} = \frac{2x-11}{x-2}$

$\Rightarrow \frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$

$\Rightarrow (2x^2 + 4)(x - 2) = (2x - 11)(x^2 + x - 2)$

$\Rightarrow 2x^3 - 4x^2 + 4x - 8 = 2x^3 - 9x^2 - 15x + 22$

$\Rightarrow \qquad 5x^2 + 19x - 30 = 0$

$\Rightarrow \qquad 5x^2 + 25x - 6x - 30 = 0$

$\Rightarrow \qquad 5x(x + 5) - 6(x + 5) = 0$

$\Rightarrow \qquad (5x - 6)(x + 5) = 0$

$\Rightarrow \qquad\qquad x = \frac{6}{5}, -5$

70. Solve for x: $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7.$

[CBSE 2020]

Ans. Given, $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$\Rightarrow \frac{x - 7 - x - 4}{(x+4)(x-7)} = \frac{11}{30}$

$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$

$\Rightarrow \frac{-1}{(x+4)(x-7)} = \frac{1}{30}$

$\Rightarrow \qquad (x + 4)(x - 7) = -30$

$\Rightarrow \qquad (x + 4)(x - 7) + 30 = 0$

$\Rightarrow x^2 + 4x - 7x - 28 + 30 = 0$

$\Rightarrow \qquad x^2 - 3x - 28 + 30 = 0$

$\Rightarrow \qquad x^2 - 3x + 2 = 0$

$\Rightarrow \qquad x^2 - 2x - x + 2 = 0$

$\Rightarrow \qquad x(x - 2) - 1(x - 2) = 0$

$\Rightarrow \qquad (x - 2)(x - 1) = 0$

i.e. $\qquad x - 1 = 0$ or $x - 2 = 0$

$\Rightarrow \qquad x = 1$ or 2

71. Determine the condition for one root of the quadratic equation $ax^2 + bx + c = 0$ to be thrice the other. [Diksha]

Ans. Let the roots of the given quadratic equation be α and 3α.

Then, sum of the roots $= \alpha + 3\alpha$

$$= 4\alpha = -\frac{b}{a}$$

$\Rightarrow \qquad \alpha = -\frac{b}{4a}$

Product of the roots $= \alpha \times 3\alpha$

$$= 3\alpha^2 = \frac{c}{a}$$

$\Rightarrow \qquad 3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a}$

$\Rightarrow \qquad \frac{3b^2}{16a^2} = \frac{c}{a}$

$\Rightarrow \qquad 3b^2 = 16ac,$

which is the required condition.

72. If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, then find the value of p and the other root of the equation. [CBSE SQP 2020]

Ans.

$$3x^2 + px + 4 = 0$$

$$3\left(\frac{2}{3}\right)^2 + p\left(\frac{2}{3}\right) + 4 = 0$$

$$\frac{4}{3} + \frac{2p}{3} + 4 = 0$$

$$p = -8$$

$$3x^2 - 8x + 4 = 0$$

$$3x^2 - 6x - 2x + 4 = 0$$

$$x = \frac{2}{3} \text{ or } x = 2$$

Hence, $\qquad x = 2$

[CBSE Marking Scheme SQP 2020]

Explanation: Given, quadratic equation is

$$3x^2 + px + 4 = 0$$

If one root of the given equation is $\frac{2}{3}$ then it will satisfy the equation.

$$3\left(\frac{2}{3}\right)^2 + p\left(\frac{2}{3}\right) + 4 = 0$$

$\Rightarrow \qquad \frac{4}{3} + \frac{2p}{3} + 4 = 0$

$\Rightarrow \qquad \frac{2p}{3} = \frac{-16}{3}$

$\Rightarrow \qquad p = -8$

$\therefore$ Quadratic equation is

$$3x^2 - 8x + 4 = 0$$

On splitting the middle term, we get

$$3x^2 - 6x - 2x + 4 = 0$$

$\Rightarrow 3x(x - 2) - 2(x - 2) = 0$

$\Rightarrow \qquad x = 2$ or $\frac{2}{3}$

Hence, the value of 'p' is -8 and other root of the equation is 2.

73. Find the value of p for which the quadratic equation $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$, $p \neq -1$, has equal roots. Hence, find the roots of the equation.

Ans. Given equation is:

$(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0 \qquad ...(i)$

Since, the equation has equal roots

$\therefore \qquad$ Discriminant $= 0$

$\Rightarrow \qquad b^2 - 4ac = 0$

Here, $a = (p + 1)$, $b = -6(p + 1)$, $c = 3(p + 9)$

$\therefore \quad [-6(p + 1)]^2 - 4 \times (p + 1) \times 3(p + 9) = 0$

$\therefore \quad (p + 1)[36(p + 1) - 12(p + 9)] = 0$

$\Rightarrow \qquad 36p + 36 - 12p - 108 = 0$

$\qquad\qquad\qquad [\because p \neq -1 \text{ (Given)}]$

$\Rightarrow \qquad 24p - 72 = 0$

$\Rightarrow \qquad p = \frac{72}{24}$

$\qquad\qquad\qquad = 3$

Putting $p = 3$ in equation (i), we get

$$4x^2 - 24x + 36 = 0$$

$\Rightarrow \qquad x^2 - 6x + 9 = 0$

$\Rightarrow \qquad (x - 3)^2 = 0$

$\Rightarrow \qquad x = 3, 3$

Hence, the value of p is 3 and the roots of the equation so formed are 3 and 3.

74. The roots α and β of the quadratic equation $x^2 - 5x + 3(k - 1) = 0$ are such that $\alpha - \beta = 1$. Find the value of k. [CBSE SQP 2020]

Ans.

$\alpha + \beta = 5 \qquad ...(i)$

$\alpha - \beta = 1 \qquad ...(ii)$

Solving (i) and (ii), we get

$\alpha = 3$ and $\beta = 2$

also $\qquad \alpha\beta = 6$

or $\qquad 3(k - 1) = 6$

$\qquad k - 1 = 2$

$\qquad k = 3$

[CBSE Marking Scheme SQP 2020]

Explanation: Given, quadratic equation is,

$$x^2 - 5x + 3(k - 1) = 0$$

and its roots are α and β, such that

$$\alpha - \beta = 1 \qquad ...(i)$$

Now sum of roots, i.e.

$$\alpha + \beta = \frac{-(-5)}{1} = 5$$

$$\therefore \qquad \alpha + \beta = 5 \qquad ...(ii)$$

On solving equations (i) and (ii), we get

$$\alpha = 3 \text{ and } \beta = 2$$

Then, product of roots, $\alpha\beta = \dfrac{3(k-1)}{1}$

$$\Rightarrow \qquad 3 \times 2 = 3k - 3$$

$$\Rightarrow \qquad 3k = 9$$

$$\Rightarrow \qquad k = 3$$

75. Ⓐ The sum of the areas of two squares is 157 m². If the sum of their perimeters is 68 m, find the sides of the two squares. **[CBSE 2019]**

76. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained. **[CBSE 2019]**

Ans. Given equation: $x^2 + px + 16 = 0$

Here, $a = 1, b = p, c = 16$

Discriminant, $D = b^2 - 4ac$

$$= p^2 - 4 \times 1 \times 16$$

$$= p^2 - 64$$

If roots are equal, then

$$D = 0$$

i.e., $\qquad p^2 - 64 = 0$

$$\Rightarrow \qquad p^2 = 64$$

$$\Rightarrow \qquad p = \pm 8$$

$\therefore$ Equation is $x^2 \pm 8x + 16 = 0$

$$\Rightarrow \qquad (x \pm 4)^2 = 0$$

$$[\because (a \pm b)^2 = a^2 \pm 2ab + b^2]$$

$$\Rightarrow \qquad x \pm 4 = 0$$

$$\Rightarrow \qquad x = -4, 4$$

Hence, roots are $x = -4$ and $x = 4$ and the values of p are -8 and 8.

77. Ⓐ The municipality decides to plant a fountain in the rectangular corner of a park. The rectangular corner has dimensions 50m x 40m. The fountain is in the middle of the rectangular corner and the area of the grass surrounding the fountain is 1184 m². Find the length and breadth of the rectangular corner. **[Delhi Gov. SQP 2022]**

78. Ⓐ A line segment AB of length 2 m is divided at a point C into two parts such that $AC^2 = AB \times CB$. Find the length of CB. **[CBSE 2017]**

79. Show that if the roots of the following quadratic equation are equal, then $ad = bc$.
$x^2(a^2 + b^2) + 2(ac + bd)x + (c^2 + d^2) = 0$ **[CBSE 2017]**

Ans. Given, quadratic equation is:

$$x^2(a^2 + b^2) + 2(ac + bd)x + (c^2 + d^2) = 0$$

Comparing the given equation with $Ax^2 + Bx + C = 0$, we get

$$A = a^2 + b^2$$

$$B = 2(ac + bd)$$

$$C = c^2 + d^2$$

Since, roots of the given equation are equal.

$\therefore$ Discriminant $= B^2 - 4AC = 0$

$$\Rightarrow \quad [2(ac + bd)]^2 - 4 \times (a^2 + b^2) \times (c^2 + d^2) = 0$$

$$\Rightarrow 4(ac + bd)^2 - 4(a^2c^2 + b^2c^2 + a^2d^2 + b^2d^2) = 0$$

$$\Rightarrow 4(a^2c^2 + b^2d^2 + 2abcd) - 4a^2c^2 - 4b^2c^2$$
$$- 4a^2d^2 - 4b^2d^2 = 0$$

$$\Rightarrow \quad 8abcd - 4a^2d^2 - 4b^2c^2 = 0$$

$$\Rightarrow \quad -4(a^2d^2 + b^2c^2 - 2abcd) = 0$$

$$\Rightarrow \quad a^2.d^2 + b^2c^2 - 2abcd) = 0$$

$$\Rightarrow \quad (ad - bc)^2 = 0$$

On taking square-root on both sides

$$\Rightarrow \qquad ad - bc = 0$$

$$\Rightarrow \qquad ad = bc$$

Hence, proved.

80. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$. **[CBSE 2017]**

Ans. Given, quadratic equation is,

$$(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$$

On comparing it with $Ax^2 + Bx + C = 0$, we get

$A = 1 + m^2$, $B = 2mc$ and $C = c^2 - a^2$

The roots of the given equation are equal, then

Discriminant, $D = 0$

$\therefore \qquad B^2 - 4AC = 0$

$$(2mc)^2 - 4 \times (1 + m^2) \times (c^2 - a^2) = 0$$

$$\Rightarrow \quad 4m^2c^2 - 4(c^2 + c^2m^2 - a^2 - a^2m^2) = 0$$

$$\Rightarrow \quad 4m^2c^2 - 4c^2 - 4c^2m^2 + 4a^2 + 4m^2a^2 = 0$$

$$\Rightarrow \qquad\qquad m^2a^2 + a^2 - c^2 = 0$$

$$\Rightarrow \qquad\qquad c^2 = m^2a^2 + a^2$$

$$\Rightarrow \qquad\qquad c^2 = a^2(1 + m^2)$$

Hence, proved.

81. In a class test, the sum of Shrishti's marks in maths and science is 45. If she had 1 more mark in maths and 1 less in science, the product of marks would have been 500. Find her marks in maths and science.

Ans. Let the Shrishti's marks in maths be x.

Then, her marks in science $= 45 - x$

Now, according to the question,

$$(x + 1)[(45 - x) - 1] = 500$$

$\Rightarrow \quad (x + 1)(44 - x) = 500$

$\Rightarrow \quad 44x + 44 - x^2 - x = 500$

$\Rightarrow \quad x^2 - 43x + 456 = 0$

$\Rightarrow \quad x^2 - 24x - 19x + 456 = 0$

$\Rightarrow \quad x(x - 24) - 19(x - 24) = 0$

$\Rightarrow \quad (x - 19)(x - 24) = 0$

$\Rightarrow \quad x = 19, 24$

If $x = 19$, then $(45 - x) = 45 - 19 = 26$

If $x = 24$, then $(45 - x) = 45 - 24 = 21$

Hence, she either got 19 marks in maths and 26 marks in science or 24 marks in maths and 21 marks in science.

82. ④ Solve the given quadratic equation for x:
$9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$
[CBSE 2016]

83. Find two consecutive odd positive integers, sum of whose squares is 290.
[CBSE SQP 2020]

Ans. Let one of the odd positive integer be x then the other odd positive integer is $x + 2$ their sum of squares $= x^2 + (x + 2)^2$

$= x^2 + x^2 + 4x + 4$

$= 2x^2 + 4x + 4$

Given that their sum of squares = 290

$\Rightarrow \quad 2x^2 + 4x + 4 = 290$

$\Rightarrow \quad 2x^2 + 4x = 290 - 4 = 286$

$\Rightarrow \quad 2x^2 + 4x - 286 = 0$

$\Rightarrow \quad 2(x^2 + 2x - 143) = 0$

$\Rightarrow \quad x^2 + 2x - 143 = 0$

$\Rightarrow \quad x^2 + 13x - 11x - 143 = 0$

$\Rightarrow \quad x(x + 13) - 11(x + 13) = 0$

$\Rightarrow \quad (x - 11)(x + 13) = 0$

$\Rightarrow \quad (x - 11) = 0, (x + 13) = 0$

Therefore, $x = 11$ or -13

According to question, x is a positive odd integer.

Hence, we take positive value of x

So, $x = 11$ and $(x + 2) = 11 + 2 = 13$

Therefore, the odd positive integers are 11 and 13.

[CBSE Marking Scheme SQP 2020]

84. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, it would have taken 3 hours more to cover the same distance. Find the original speed of the train. [CBSE 2020]

Ans. Let the original speed of the train be x km/h.

Then, time of journey of 480 km $= \dfrac{480}{x}$ hours

Time taken to cover a journey of 480 km with speed of $(x - 8)$ km/h $= \dfrac{480}{x-8}$ hours

Now, according to question,

$$\dfrac{480}{x-8} - \dfrac{480}{x} = 3$$

$\Rightarrow \quad 480\left[\dfrac{x-x+8}{x(x-8)}\right] = 3$

$\Rightarrow \quad 3x(x - 8) = 3840$

$\Rightarrow \quad x(x - 8) = 1280$

$\Rightarrow \quad x^2 - 8x - 1280 = 0$

$\Rightarrow \quad x^2 - 40x + 32x - 1280 = 0$

$\Rightarrow \quad x(x - 40) + 32(x - 40) = 0$

$\Rightarrow \quad (x + 32)(x - 40) = 0$

$\Rightarrow \quad x + 32 = 0$ or $x - 40 = 0$

$\because \qquad x = -32$ (not possible)

$\therefore \qquad x = 40$

Thus, the original speed of the train is 40 km/h.

85. ④ Find the value of k for which the quadratic equation $kx^2 + 1 - 2(k - 1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.
[CBSE 2020]

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

86. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train. [CBSE 2019]

Ans. Let, the actual speed of the train be 'x' km/hr.

So, time taken by the train at actual speed,

$$t_1 = \dfrac{360}{x} \text{hr}$$

Increased speed of the train $= (x + 5)$ km/hr

So, time taken by train at the increased speed,

$$t_2 = \dfrac{360}{x+5} \text{hr}$$

According to the given condition:

$$t_1 - t_2 = 1$$

$\Rightarrow \quad \dfrac{360}{x} - \dfrac{360}{x+5} = 1$

$\Rightarrow \quad \dfrac{360(x + 5 - x)}{(x+5)x} = 1$

$\Rightarrow$

$\Rightarrow \quad 360 \times 5 = x^2 + 5x$

$x^2 + 5x - 1800 = 0$

$\Rightarrow x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow x(x + 45) - 40(x + 45) = 0$

$\Rightarrow (x - 40)(x + 45) = 0$

$\Rightarrow x = 40$

[∵ $x = -45$, is not possible as speed cannot be negative]

Hence, the actual speed of the train is 40 km/hr.

87. Solve for x: $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$;

$a \neq b \neq 0, x \neq 0, x \neq -(a+b)$ **[CBSE 2019]**

Ans. Given, $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$

$\Rightarrow \dfrac{1}{a+b+x} - \dfrac{1}{x} = \dfrac{1}{a} + \dfrac{1}{b}$

$\Rightarrow \dfrac{x - a - b - x}{x(a+b+x)} = \dfrac{a+b}{ab}$

$\Rightarrow \dfrac{-(a+b)}{x(a+b+x)} = \dfrac{(a+b)}{ab}$

$\Rightarrow -ab = ax + bx + x^2$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow x(x + a) + b(x + a) = 0$

$\Rightarrow (x + b)(x + a) = 0$

$\Rightarrow x = -a, -b$

Hence, the values of x are $-a$ and $-b$.

88. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.   **[NCERT Exemplar]**

Ans. Let n be the required natural number.

Square of natural number diminished by 84 gives $n^2 - 84$.

Thrice of 8 more than given number $= 3(8 + n)$.

According to the question,

$n^2 - 84 = 3(8 + n)$

$\Rightarrow n^2 - 84 = 24 + 3n$

$\Rightarrow n^2 - 3n - 108 = 0$

Splitting the middle term, we have

$\Rightarrow n^2 - 12n + 9n - 108 = 0$

$\Rightarrow n(n - 12) + 9(n - 12) = 0$

$\Rightarrow (n - 12)(n + 9) = 0$

$n = 12$ or $n = -9$

But $n \neq -9$, as n is a natural number.

Hence, the required natural number is 12.

89. ✎ **A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.**   **[Delhi Gov. Q.B. 2022, CBSE 2014]**

90. ✎ **A plane left 30 minutes later than the scheduled time and in order to reach its destination 1500 km away on time, it has to increase its speed by 250 km/hr from its usual speed. Find the usual speed of the plane.**   **[CBSE 2019]**

91. Find the dimensions of a rectangular park whose perimeter is 60 m and area 200 m².   **[CBSE 2019]**

Ans. Let 'l' be the length and 'b' be the breadth of the rectangular park.

Perimeter of the park, $P = 2(l + b)$

Area of the park, $A = l \times b$

According to the given conditions:

$2(l + b) = 60$

$\Rightarrow l + b = 30$

$\Rightarrow l = 30 - b$ ...(i)

and $lb = 200$

$\Rightarrow (30 - b)b = 200$ [from (i)]

$\Rightarrow 30b - b^2 = 200$

$\Rightarrow b^2 - 30b - 200 = 0$

On splitting the middle term, we get

$\Rightarrow b^2 - 20b - 10b + 200 = 0$

$\Rightarrow b(b - 20) - 10(b - 20) = 0$

$\Rightarrow (b - 20)(b - 10) = 0$

$\Rightarrow b = 20$ or 10

When, $b = 20$ m, $l = 10$ m

When, $b = 10$ m, $l = 20$ m

Hence, the length and breadth of the rectangular park, are 10 m and 20 m or 20 m and 10 m respectively.

92. To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool?   **[CBSE SQP Std. 2022]**

Ans. Let the time taken by larger pipe alone to fill the tank $= x$ hours

Therefore, the time taken by the smaller pipe $= x + 10$ hours

Water filled by larger pipe running for 4 hours

$= \dfrac{4}{x}$ litres

Water filled by smaller pipe running for 9 hours

$= \dfrac{9}{x+10}$ litres

We know that, $\dfrac{4}{x} + \dfrac{9}{x+10} = \dfrac{1}{2}$

Which on simplification gives:

$x^2 - 16x - 80 = 0$

$x^2 - 20x + 4x - 80 = 0$

$x(x - 20) + 4(x - 20) = 0$

$(x + 4)(x - 20) = 0$

$x = -4, 20$

x cannot be negative.

Thus, $x = 20$
                          $x + 10 = 30$

Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours.

[CBSE Marking Scheme SQP Std. 2022]

93. At present, Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.

[CBSE 2010, NCERT Exemplar]

94. In a class test, the sum of Arun's marks in Hindi and English is 30. Had he got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

[CBSE 2019]

95. A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

[CBSE SQP 2020, 18]

Ans. Let, the speed of the stream be x km/hr.

Speed of the boat in still water = 18 km/hr

∴ The speed of the boat in upstream
                          $= (18 - x)$ km/hr

The speed of the boat in downstream
                          $= (18 + x)$ km/hr

Total distance to be covered $= 24$ km

∴ Time taken in upstream, $t_1 = \dfrac{24}{(18 - x)}$ hr

Time taken in downstream, $t_2 = \dfrac{24}{(18 + x)}$ hr

According to the question,

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1$$

$$\Rightarrow \frac{24(18 + x - 18 + x)}{(18 - x)(18 + x)} = 1$$

$$\Rightarrow \qquad 24 \times 2x = 324 - x^2$$
$$\Rightarrow \qquad x^2 + 48x - 324 = 0$$
$$\Rightarrow \qquad x^2 + 54x - 6x - 324 = 0$$
$$\Rightarrow \qquad (x + 54)(x - 6) = 0$$
$$\qquad\qquad x \neq -54$$

(∵ speed can't be negative)

∴ $x = 6$

Hence, the speed of the stream is 6 km/hr.

96. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed?

[CBSE 2013]

Ans. Let the original speed of the train be 'x' km/hr.

∴ Increased speed = $(x + 6)$ km/hr

Now, time taken to cover 63 km at original speed, $t_1 = \dfrac{63}{x}$ hr

Time taken to cover 72 km at increased speed,

$$t_2 = \frac{72}{x + 6} \text{ hr}$$

According to question,

$$\frac{63}{x} + \frac{72}{x + 6} = 3$$

$$\Rightarrow \frac{63x + 378 + 72x}{x(x + 6)} = 3$$

$$\Rightarrow \qquad 135x + 378 = 3(x^2 + 6x)$$
$$\Rightarrow \qquad 3x^2 + 18x - 135x - 378 = 0$$
$$\Rightarrow \qquad 3x^2 - 117x - 378 = 0$$
$$\Rightarrow \qquad x^2 - 39x - 126 = 0$$
$$\Rightarrow \qquad x^2 - 42x + 3x - 126 = 0$$
$$\Rightarrow \qquad x(x - 42) + 3(x - 42) = 0$$
$$\Rightarrow \qquad (x - 42)(x + 3) = 0$$
$$\qquad\qquad x \neq -3$$

(speed can't be negative)

∴ $x = 42$

Hence, the original average speed of train is 42 km/hr.

97. In a flight of 600km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr from its usual speed and the time of the flight increased by 30 min. Find the scheduled duration of the flight. **[CBSE SQP Std. 2022]**

Ans. Let the usual speed of plane be x km/hr and the reduced speed of the plane be (x − 200) km/hr

Distance = 600 km [Given]

According to the question,

(time taken at reduced speed) − (Schedule time)
                          = 30 minutes = 0.5 hours

$$\frac{600}{x - 200} - \frac{600}{x} = \frac{1}{2}$$

Which on simplification gives:
$$x^2 - 200x - 240000 = 0$$
$$x^2 - 600x + 400x - 240000 = 0$$
$$x(x - 600) + 400(x - 600) = 0$$
$$(x - 600)(x + 400) = 0$$
$$x = 600 \text{ or } x = -400$$

But speed cannot be negative.

∴ The usual speed is 600 km/hr

and the scheduled duration of the flight is

$$\frac{600}{600} = 1 \text{ hour}$$

[CBSE Marking Scheme SQP Std. 2022]

98. Find two consecutive odd natural numbers, the sum of whose squares is 394.

[CBSE 2017]

99. A and B working together can do a work in 6 days. If A takes 5 days less than B to finish the work, in how many days can B do the same work alone?

Ans. Let B take 'x' days to complete the work alone. Then, A takes $(x - 5)$ days to complete the work done.

According to the given condition:

$$\frac{1}{x} + \frac{1}{x-5} = \frac{1}{6}$$

$$\Rightarrow \qquad \frac{x-5+x}{x(x-5)} = \frac{1}{6}$$

$$\Rightarrow \qquad 6(2x-5) = x^2 - 5x$$

$$\Rightarrow \qquad x^2 - 5x - 12x + 30 = 0$$

$$\Rightarrow \qquad x^2 - 17x + 30 = 0$$

$$\Rightarrow \qquad x^2 - 15x - 2x + 30 = 0$$

On splitting the middle term

$$\Rightarrow \qquad x(x-15) - 2(x-15) = 0$$

$$\Rightarrow \qquad (x-15)(x-2) = 0$$

$$\Rightarrow \qquad x = 2 \text{ or } 15$$

But $x = 2$ is not possible as, then, $(x - 5)$ would be negative.

$$\therefore \qquad x = 15$$

Hence, B takes 15 days to complete the work alone.

100. Find x in terms of a, b and c:

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c \qquad \text{[CBSE 2016]}$$

101. At t minutes past 2 pm, the time needed by the minute hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t. [NCERT Exemplar]

Ans. It is given that at t minutes past 2 pm, the time needed by the minute hand to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ min.

$$\Rightarrow \qquad t + \left(\frac{t^2}{4} - 3\right) = 60$$

[∵ time between 2 pm and 3 pm = 1 hour = 60 min]

$$\Rightarrow \qquad 4t + t^2 - 12 = 240$$

$$\Rightarrow \qquad t^2 + 4t - 12 - 240 = 0$$

$$\Rightarrow \qquad t^2 + 4t - 252 = 0$$

Splitting the middle term, we have

$$t^2 + 18t - 14t - 252 = 0$$

$$\Rightarrow t(t+18) - 14(t+18) = 0$$

$$\Rightarrow \qquad (t+18)(t-14) = 0$$

$$t = -18 \text{ or } t = 14.$$

But $t \neq -18$ as time cannot be negative

$$\Rightarrow \qquad t = 14$$

Hence, the required value of t is 14 minutes.

102. Solve for x:

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0,$$

$$x \neq 3, -\frac{3}{2}$$

[CBSE 2016]

Ans.

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$$

$$\Rightarrow \qquad \frac{2x(2x+3) + (x-3) + 3x + 9}{(x-3)(2x+3)} = 0$$

$$\Rightarrow \qquad 4x^2 + 6x + x - 3 + 3x + 9 = 0$$

$$\Rightarrow \qquad 4x^2 + 10x + 6 = 0$$

$$\Rightarrow \qquad 2x^2 + 5x + 3 = 0$$

$$\Rightarrow \qquad 2x^2 + 2x + 3x + 3 = 0$$

(on splitting the middle term)

$$\Rightarrow \qquad 2x(x+1) + 3(x+1) = 0$$

$$\Rightarrow \qquad (2x+3)(x+1) = 0$$

$$\Rightarrow \qquad x = \frac{-3}{2} \text{ or } -1$$

But $\qquad x = -\frac{3}{2}$ (Given)

Hence, the value of x is -1.

103. Sum of the areas of two squares is 544 m^2. If the difference of their perimeters is 32 find the sides of the two squares.

[CBSE 2020]

104. In a flight of 600 km, the speed of the aircraft was slowed down due to bad weather. The average speed of the trip was decreased by 200 km/hr and thus the time of flight increased by 30 minutes. Find the average speed of the aircraft originally.

[CBSE 2020]

105. ₹9,000 were divided equally among a certain number of persons. Had there been 20 more persons, each would have got ₹ 160 less. Find the original number of persons.

[CBSE 2020]

Ans. Let the original number of persons be 'x', then,

each person gets ₹$\left(\frac{9000}{x}\right)$.

When the number of persons is '$x + 20$', then,

each person gets ₹$\left(\frac{9000}{x+20}\right)$.

As per the question,

$$\frac{9000}{x} - \frac{9000}{x+20} = 160$$

$$\Rightarrow \quad 9000\left[\frac{x+20-x}{x(x+20)}\right] = 160$$

$$\Rightarrow \quad 160x(x+20) = 9000 \times 20$$

$$\Rightarrow \quad x(x+20) = 1125$$

$$\Rightarrow \quad x(x+20) - 1125 = 0$$

$$\Rightarrow \quad x^2 + 20x - 1125 = 0$$

$$\Rightarrow x^2 + 45x - 25x - 1125 = 0$$

$$\Rightarrow \quad x(x+45) - 25(x+45) = 0$$

$$\Rightarrow \quad (x+45)(x-25) = 0$$

$$\Rightarrow \quad x-25 = 0 \quad (\because x+45 \neq 0)$$

$$\Rightarrow \quad x = 25$$

Thus, originally the number of persons is 25.

106. A train covers a distance of 360 km at a uniform speed. Had the speed been 5 km/hour more, it would have taken 48 minutes less for the journey. Find the original speed of the train. [CBSE SQP 2019]

Ans. Let the original speed of the train be x km/h.

$$\therefore \quad \frac{360}{x} - \frac{360}{x+5} = 48$$

$$\Rightarrow \quad x^2 + 5x - 2250 = 0$$

$$\Rightarrow \quad (x+50)(x-45) = 0 \quad \therefore x = 45$$

Hence original speed of the train = 45 km/h

[CBSE Marking Scheme SQP 2019]

Explanation: Let the original speed of the train be x km/h.

$\therefore$ Increased speed = $(x+5)$ km/h.

So, time taken to cover 360 km at original speed,

$$t_1 = \frac{360}{x}\text{hr}$$

And, time taken to cover 360 km at increased speed,

$$t_2 = \frac{360}{x+5}\text{hr}$$

Now, according to the question,

$$t_1 - t_2 = \frac{48}{60}\text{hr}$$

$$\left[\because 48\,\text{min.} = \frac{48}{60}\text{hr}\right]$$

$$\Rightarrow \quad \frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$$\Rightarrow \quad 360\left[\frac{x+5-x}{x(x+5)}\right] = \frac{4}{5}$$

$$\Rightarrow \quad 4x(x+5) = 360 \times 5 \times 5$$

$$\Rightarrow \quad x(x+5) = 2250$$

$$\Rightarrow \quad x^2 + 5x - 2250 = 0$$

$$\Rightarrow \quad x^2 + 50x - 45x - 2250 = 0$$

$$\Rightarrow \quad x(x+50) - 45(x+50) = 0$$

$$\Rightarrow \quad (x-45)(x+50) = 0$$

$$\Rightarrow \quad x - 45 = 0$$

$[\because x+50 \neq 0$, as speed cannot be negative]

$$\Rightarrow \quad x = 45$$

Hence, the original speed of the train is 45 km/h.

107. A wall shown below measure 5 m in length and 4 m in height. The outer portion of the wall of uniform width 'x' m will be painted and the central portion will be tiled. The total budget, including the tiles at ₹ 500 per m² and paint at ₹ 200 per m² is ₹ 5800.
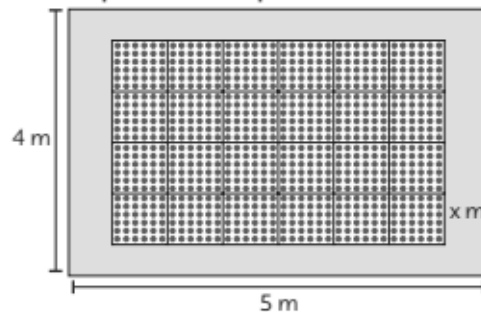
(Note: The figure is not to scale.):
Find x such that the work is completed as per the budget. Shown your work.

[CBSE Question Bank 2023]

Ans. Given, length of the wall = 5m
height of the wall = 4m

Now, the length of the tiled portion = $(5-2x)$ m
and height of the tiled portion = $(4-2x)$m

$\therefore$ Area of the tiled portion

$$= (5-2x)(4-2x)\,\text{m}^2$$

And area of the wall = 5×4

$$= 20\,\text{m}^2$$

Now, area of the painted portion

$$= [20 - (5-2x)(4-2x)]\,\text{m}^2$$

According to the question,

$$500[(5-2x)(4-2x)] + 200[20 - (5-2x)(4-2x)] = 5800$$

$$500[20 - 8x - 10x + 4x^2] + 200[20 - 20 + 8x + 10x - 4x^2] = 5800$$

$$12x^2 - 54x + 42 = 0$$

$$2x^2 - 9x + 7 = 0$$

$$2x^2 - 7x - 2x + 7 = 0$$

$$x(2x-7) - 1(2x-7) = 0$$

$$(2x-7)(x-1) = 0$$

$$x = 1, 3.5$$

$\Rightarrow x = 1$ [$x \neq 3.5$, because the painted portion would exceed the length and height of the wall]

108. Ⓐ Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with a larger diameter takes 2 hours less than the tap with the smaller one to fill the tank separately. Find the time in which each tap can fill the tank.

[CBSE 2019]

109. Two water taps together can fill a tank in $22\frac{2}{9}$ minutes. The tap of larger diameter takes 10 minutes less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

[CBSE 2016]

Ans. Let, the time taken by smaller tap be x minutes. Then, time taken by larger tap is $(x - 10)$ minutes.

Then, smaller tap's 1 minute work = $\frac{1}{x}$

Then, larger tap's 1 minute work = $\frac{1}{x-10}$

According to the question,

$$\frac{1}{x} + \frac{1}{x-10} = \frac{1}{22\frac{2}{9}}$$

$\Rightarrow \qquad \dfrac{x-10+x}{x(x-10)} = \dfrac{9}{200}$

$\Rightarrow \qquad \dfrac{2x-10}{x^2-10x} = \dfrac{9}{200}$

$\Rightarrow \qquad 200(2x - 10) = 9(x^2 - 10x)$

$\Rightarrow \qquad 400x - 2000 = 9x^2 - 90x$

$\Rightarrow 9x^2 - 90x - 400x + 2000 = 0$

$\Rightarrow \qquad 9x^2 - 490x + 2000 = 0$

$\Rightarrow 9x^2 - 450x - 40x + 2000 = 0$

$\Rightarrow \qquad 9x(x - 50) - 40(x - 50) = 0$

$\Rightarrow \qquad (9x - 40)(x - 50) = 0$

$\Rightarrow \qquad x = 50 \text{ or } x = \frac{40}{9} \text{ (rejected)}$

So, time taken by tap of smaller diameter to fill the tank = 50 minutes.

Time taken by the tap of larger diameter to fill the tank = (50 – 10) minutes = 40 minutes.

Hence, the time taken by larger and smaller taps are 40 minutes and 50 minutes respectively.